Comparison of response functions in `kitagawa`

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Abstract

In this vignette we demonstrate the response functions found in the package `kitagawa`, which are appropriate for modeling the effect of harmonic volumetric strain or pressure-head fluctuations in sealed and open water wells. For sealed-wells there is only one response function, from Kitagawa et al. (2011), and this gives the complex frequency response of virtual water height Z or pressure P as a function of areal strain ϵ. For open wells there is a suite of open-well response functions, from Cooper et al. (1965); Hsieh et al. (1987); Rojstaczer (1988); Liu et al. (1989); Wang et al. (2018); and these give the complex frequency response of water height as a function of aquifer head H or pressure. Wang et al. (2018) allows for leakage from the aquifer.

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1 Introduction

The underlying physical model of these response functions is based upon the assumption that fluid flows radially through an homogeneous, isotropic, confined aquifer. The underlying principle is as follows. When a harmonic wave induces strain in a confined aquifer (one having aquitards above and below it), fluid flows radially into, and out of a well penetrating the aquifer. The flow-induced drawdown, s, is governed by the following partial differential equation, expressed in radial coordinates(r):

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{S}{T}\frac{\partial s}{\partial t} = 0 \tag{1}$$

where S and T are the aquifer storativity and transmissivity respectively.
The solution to this PDE, with periodic discharge boundary conditions, gives the amplitude and phase response we wish to calculate. The solution for an open well was first presented by Cooper et al. (1965), and subsequently modified by Rojstaczer (1988); Liu et al. (1989). Kitagawa et al. (2011) adapted the solution of Hsieh et al. (1987) for the case of a sealed well. Wang et al. (2018) provides the leaky aquifer response for an open well.
These models are applicable to any quasi-static process involving harmonic, volumetric strain of an aquifer (e.g., passing Rayleigh waves, or changes in the Earth's tidal potential). In practice, however, the presence of permeable fractures can violate the assumption of isotropic permeability, which may substantially alter the response by introducing shear-strain coupling. Such complications are beyond the scope of these models.

2 Preliminaries

Load the necessary packages:

```
library(RColorBrewer)
Set1 <- brewer.pal(8, "Set1")
library(signal, warn.conflicts = FALSE)
library(kitagawa)

## Loaded kitagawa (3.1) - Spectral response of water wells; see ?kitagawa
```

Setup some constants:

```
S. <- 1e-05  # Storativity [nondimensional]
T. <- 1e-04  # Transmissivity [m**2 / s]
D. <- T./S.  # Diffusivity [m**2 / s]
Ta <- 50  # Aquifer thickness [m] #100
Hw <- z <- 50  # Depth to water table [m] #10

# Using AN01 stats from Kit Tbl 1
Rc. <- 0.075  # Radius of cased portion of well [m]
Lc. <- 570  # Length of cased portion of well [m]
Rs. <- 0.135  # Radius of screened portion of well [m]
Ls. <- 15  # Length of screened portion of well [m]
Vw. <- sensing_volume(Rc., Lc., Rs., Ls.)  # volume of fluid [m**3]
# parameters assumed by well_response: rho=1000 # density of rock
# [kg/m**3] Kf=2.2e9 # Bulk modulus of fluid [Pascals] grav=9.81 #
# gravitational acceleration [m/s**2]
rhog <- 9.81 * 1000
# Kitagawa Fig 7: Ku B / Kw Aw = 3 => Aw==4.8 at 40GPa
Ku. <- 4e+10  # Bulk modulus [Pascals]
B. <- 0.5  # Skemptons ratio [nondimensional]
```

And create the dimensionless frequencies, defined by $z^2\omega/2D$, where D is the hydraulic diffusivity:

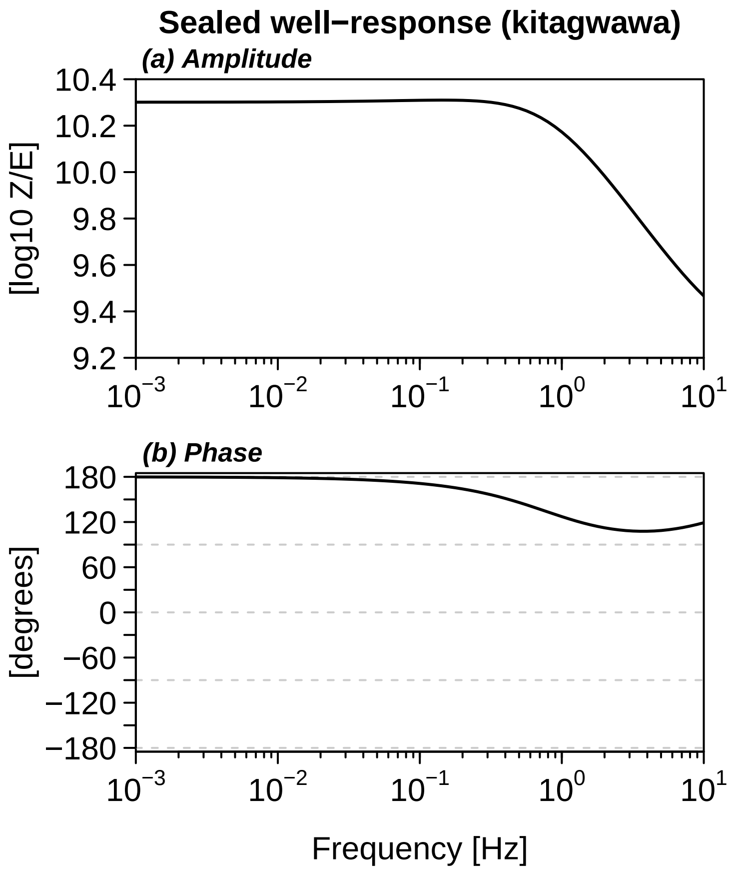
```
# Frequencies
Q <- 10^seq(-5, 2, by = 0.05)  # [nondimensional]
lQ <- log10(Q)
omega <- omega_norm(Q, z, D., invert = TRUE)  # [Hz]

Phase <- function(Z) {
    Phs. <- Arg(Z)  # will wrap to -pi/pi
    uPhs. <- signal::unwrap(Phs., tol = pi/30)
    return(data.frame(Phs = Phs., uPhs = uPhs.))
}

# Responses converted to pressure if TRUE
asP <- FALSE
ZasP <- FALSE
```

And onto the response functions...

3 Sealed well response

3.1 Strain: Kitagawa et al. (2011)

```
wrsp <- well_response(omega, T. = T., S. = S., Vw. = Vw., Rs. = Rs., Ku. = Ku.,
    B. = B., Avs = 1, Aw = 1, as.pressure = asP)
plot(wrsp)  # uses plot.wrsp method
```




```
crsp <- wrsp[["Response"]][, 2]  # Complex response
kGain <- Mod(crsp)/Ku./B.  # Amplitude (or Gain)
kP <- Phase(crsp)  # Phase
```

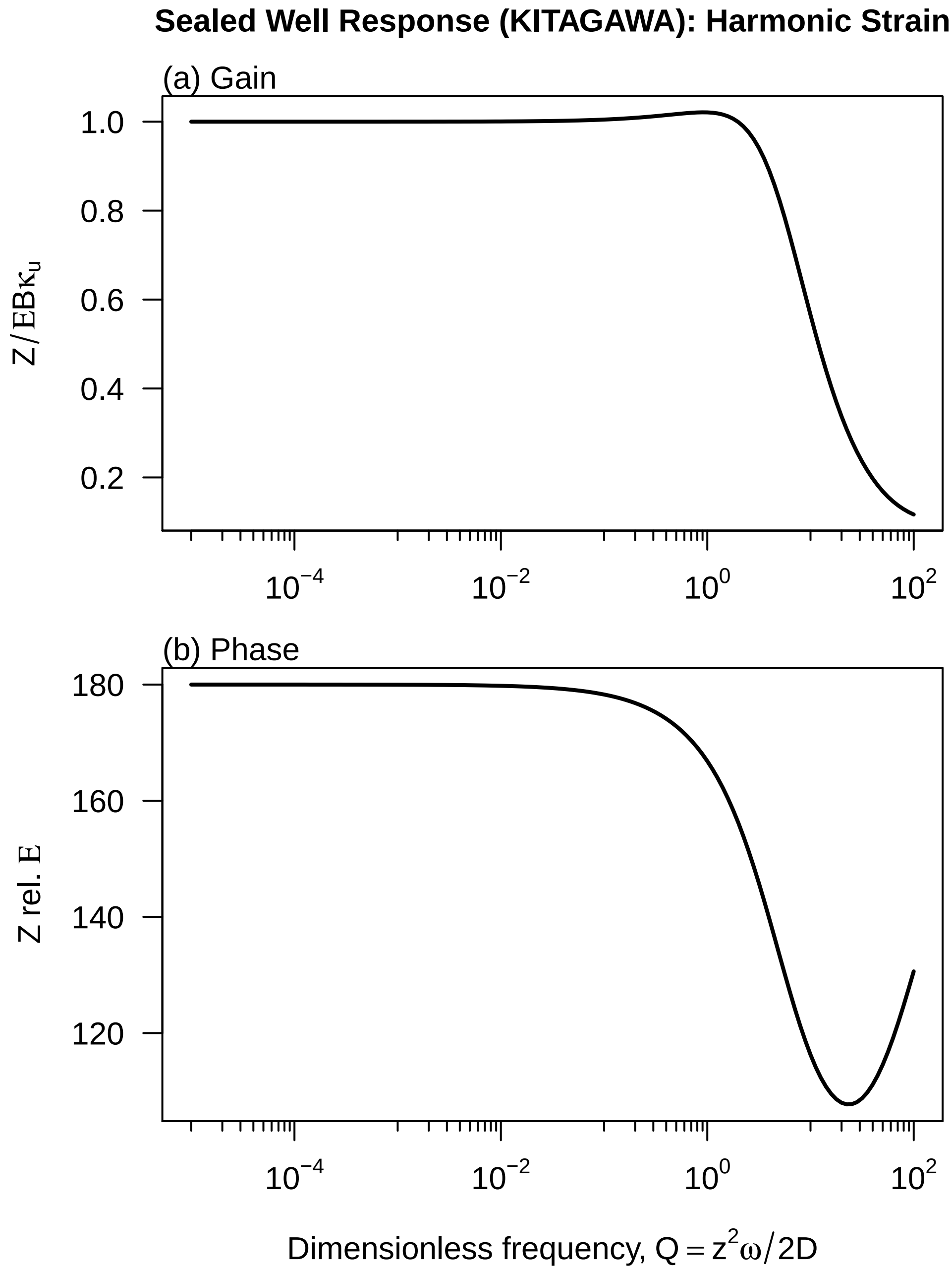
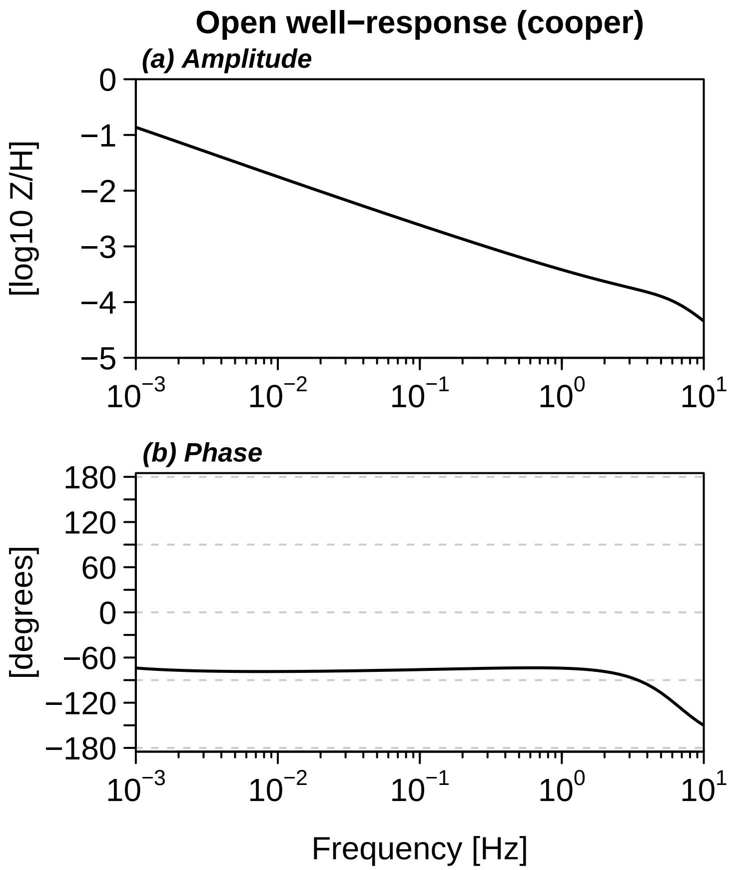


Figure 1: The response of a sealed well to harmonic areal strain using the Kitagawa model. The amplitude is normalized by Skempton's coefficient B and the undrained bulk modulus κ_u. Frequency is dimensionless, based on the well-depth z and the diffusivity D.

4 Open well response

4.1 Pressure head: Cooper et al. (1965)

```
wrsp <- open_well_response(omega, T. = T., S. = S., Ta = Ta, Hw = Hw,
    model = "cooper", as.pressure = ZasP)
plot(wrsp)
```




```
crsp <- wrsp[["Response"]][, 2]
cGain <- Mod(crsp)
cP <- Phase(crsp)
```

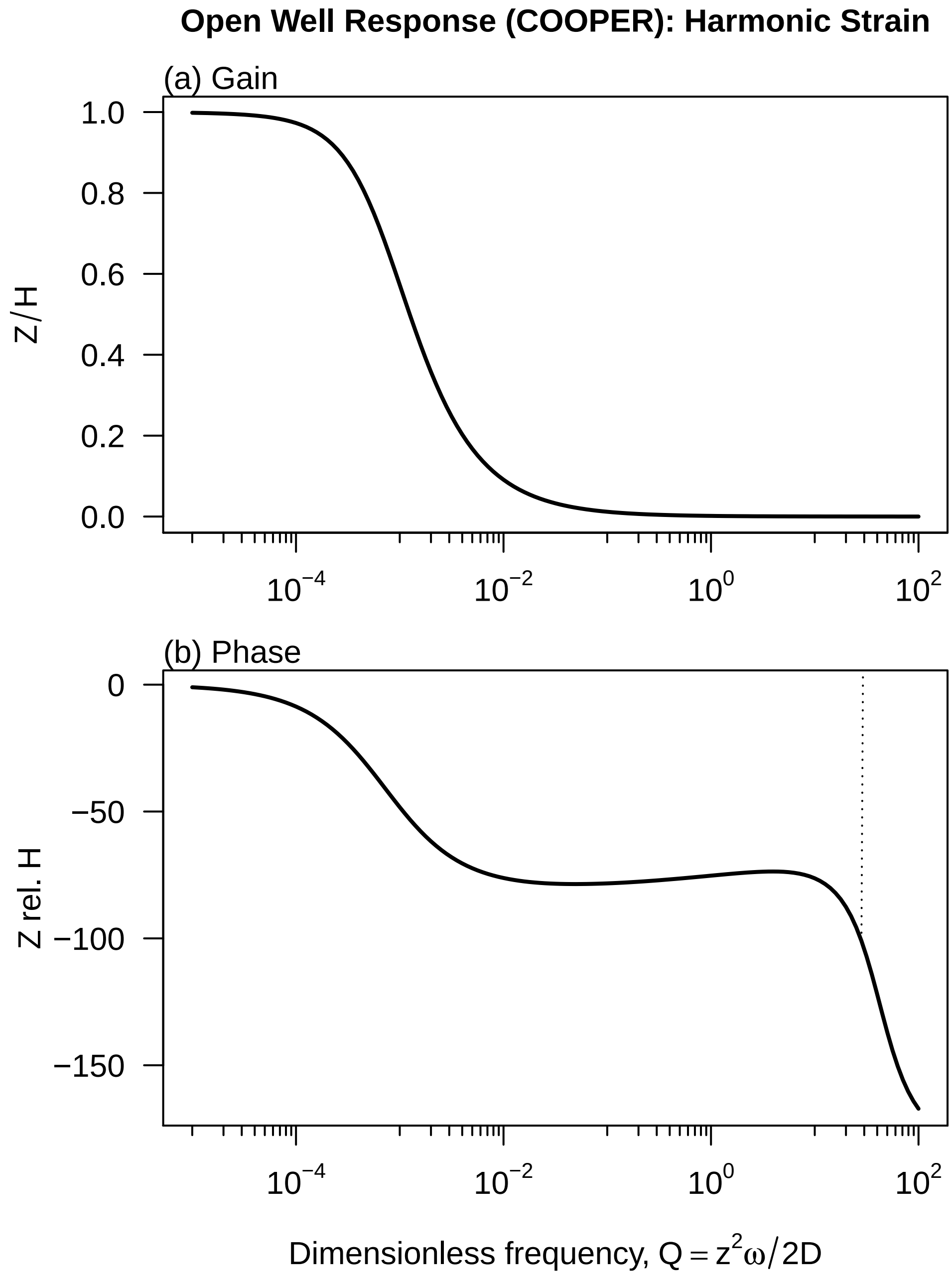
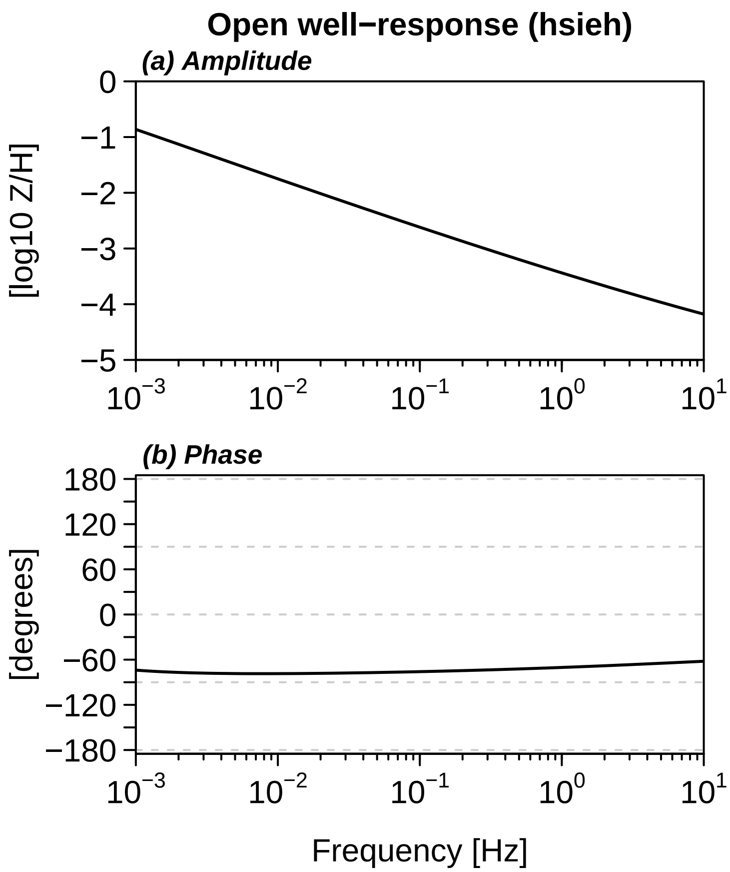


Figure 2: The response of an open well to harmonic areal strain using the Cooper model. Frequency is dimensionless, based on the well-depth z and the diffusivity D.

4.2 Pressure head: Hsieh et al. (1987)

```
wrsp <- open_well_response(omega, T. = T., S. = S., Ta = Ta, Hw = Hw,
    model = "hsieh", as.pressure = ZasP)
plot(wrsp)
```




```
crsp <- wrsp[["Response"]][, 2]
hGain <- Mod(crsp)
hP <- Phase(crsp)
```

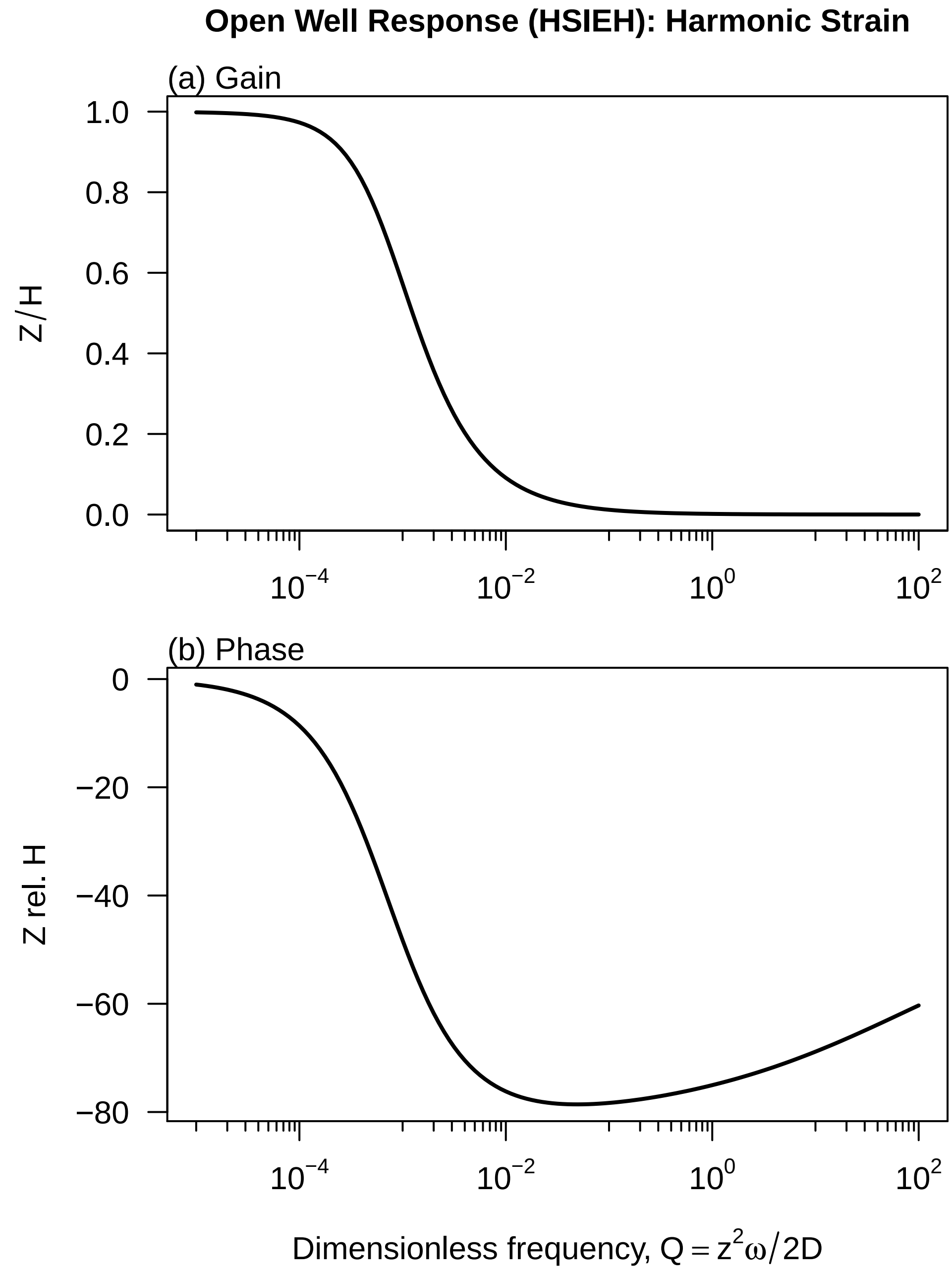
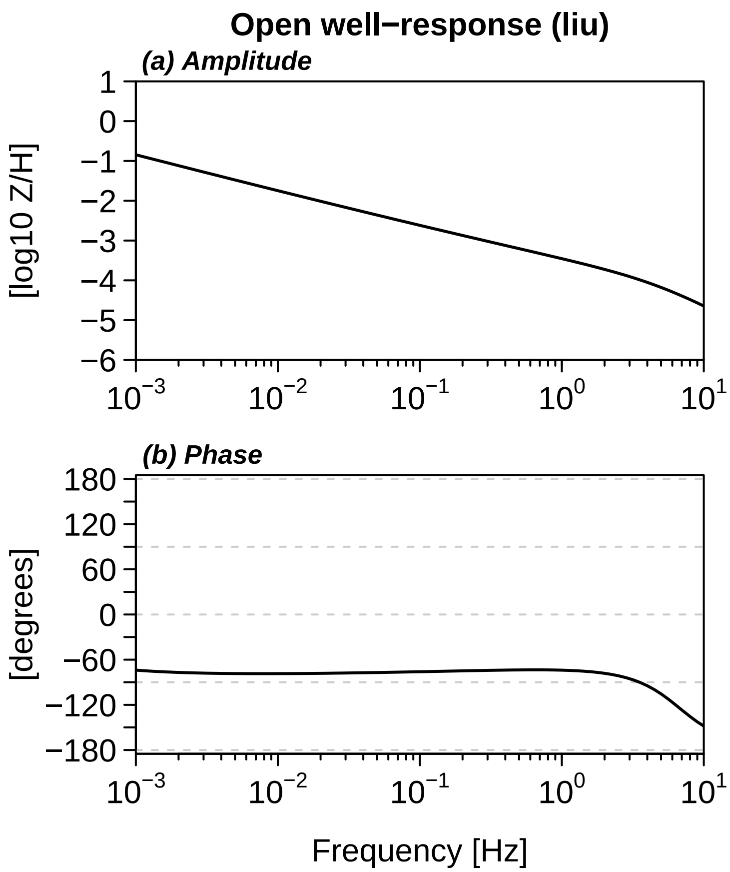


Figure 3: The response of an open well to harmonic areal strain using the Hsieh model. Frequency is dimensionless, based on the well-depth z and the diffusivity D.

4.3 Pressure head: Liu et al. (1989)

```
wrsp <- open_well_response(omega, T. = T., S. = S., Ta = Ta, Hw = Hw,
    model = "liu", as.pressure = ZasP)
plot(wrsp)
```




```
crsp <- wrsp[["Response"]][, 2]
lGain <- Mod(crsp)
lP <- Phase(crsp)
```

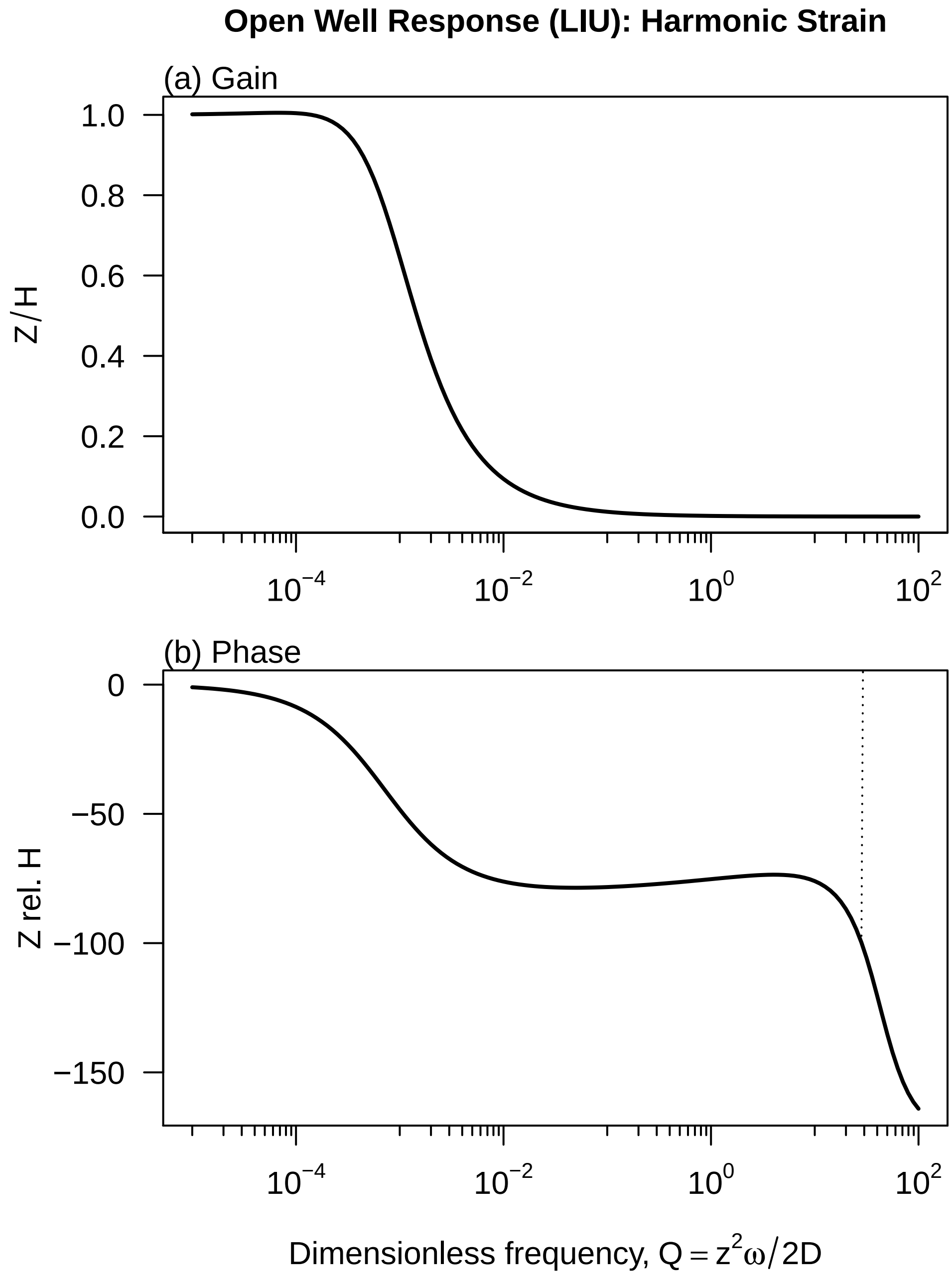
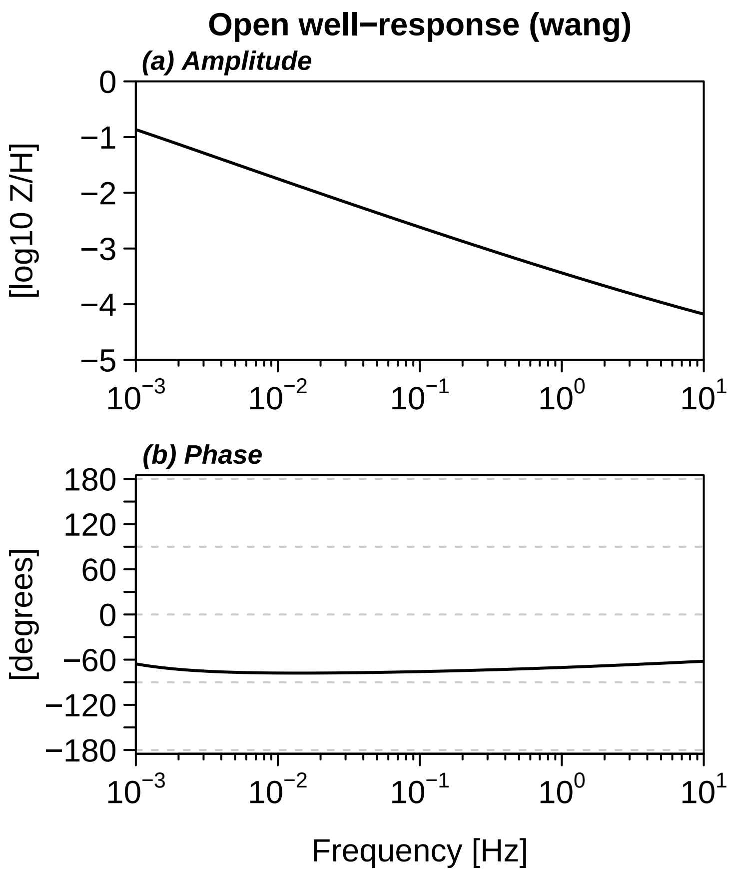


Figure 4: The response of an open well to harmonic areal strain using the Liu model. Frequency is dimensionless, based on the well-depth z and the diffusivity D.

4.4 Pressure head (with leakage): Wang et al. (2018)

```
wrsp <- open_well_response(omega, T. = T., S. = S., leak = 1e-08, model = "wang",
    as.pressure = asP)
plot(wrsp)
```




```
crsp <- wrsp[["Response"]][, 2]
rGain <- Mod(crsp)
rP <- Phase(crsp)
```

4.4.1 Figure 2 from Wang et al. (2018)

```
Transmiss <- c(1, 0.01, 1e-04, 1e-06, 1e-08)
Storativ <- c(0.01, 1e-04, 1e-06, 1e-08)
omeg <- 1.9322736/86400   # M2 in Hz
leak <- 10^seq(-11, -3, 0.2)
```

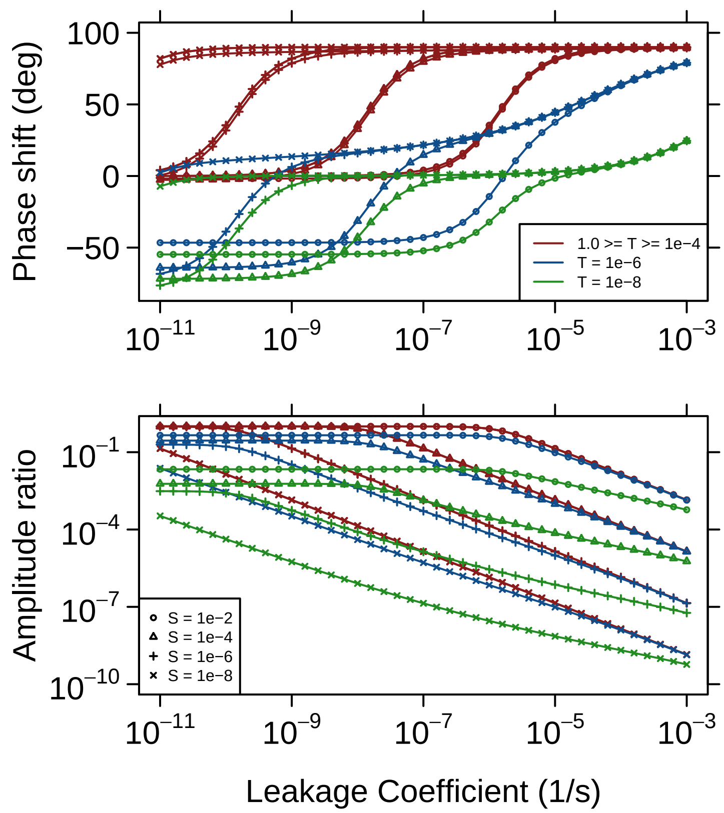
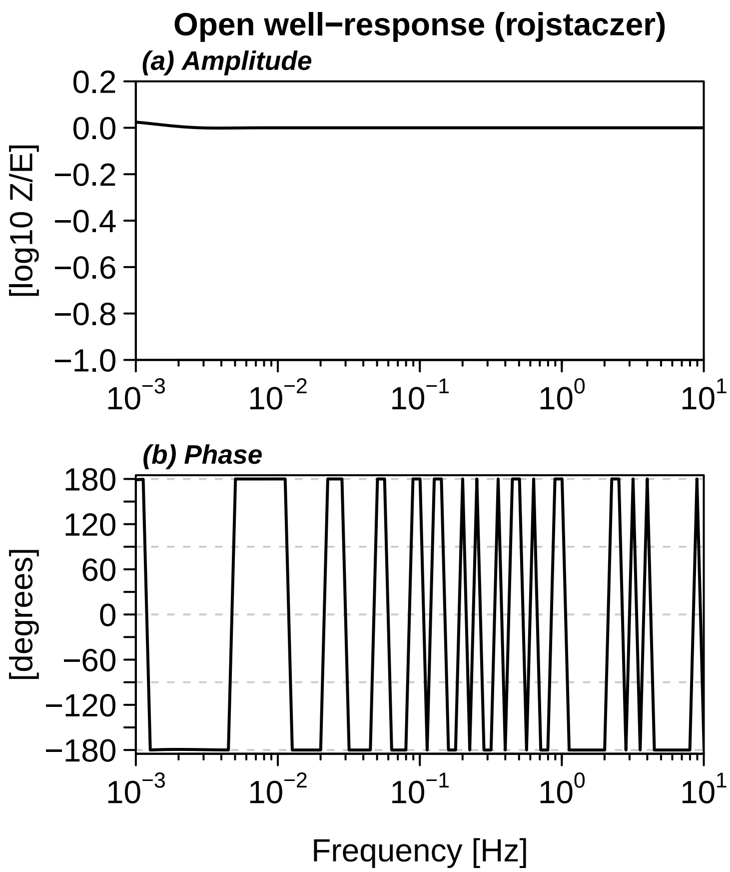


Figure 5: Amplitude and phase shift as a function of the specific leakage (K'/b') using the Wang 2018 model for the M2 tide.

4.5 Strain: Rojstaczer (1988)

```
wrsp <- open_well_response(omega, T. = T., S. = S., z = z, model = "rojstaczer",
    as.pressure = asP)
plot(wrsp)
```




```
crsp <- wrsp[["Response"]][, 2]
rGain <- Mod(crsp)
rP <- Phase(crsp)
```

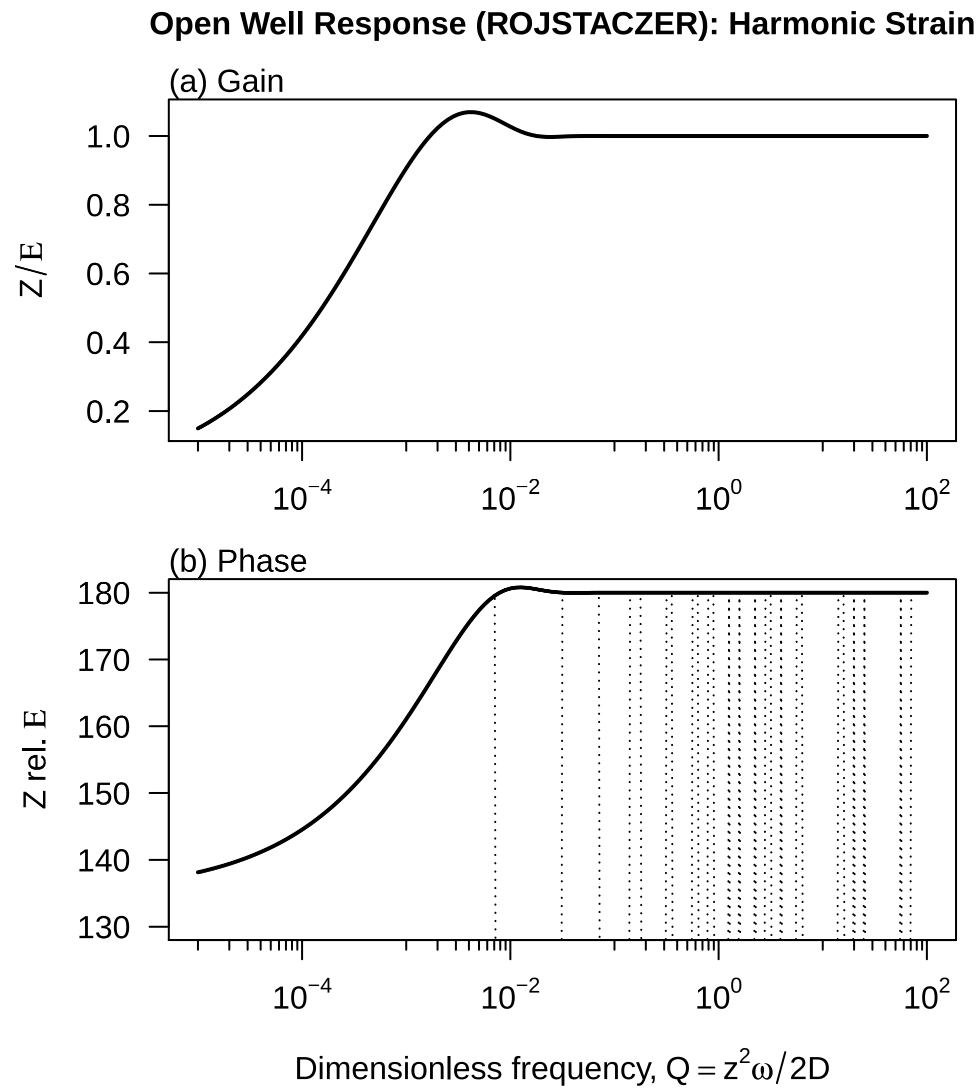


Figure 6: The response of an open well to harmonic areal strain using the Rojstaczer model. In the phase curve, phase wrapping has been removed. Modified from Rojstaczer (1988, Fig. 3). Frequency is dimensionless, based on the well-depth z and the diffusivity D.

5 Model Comparisons

5.1 Responses to strain

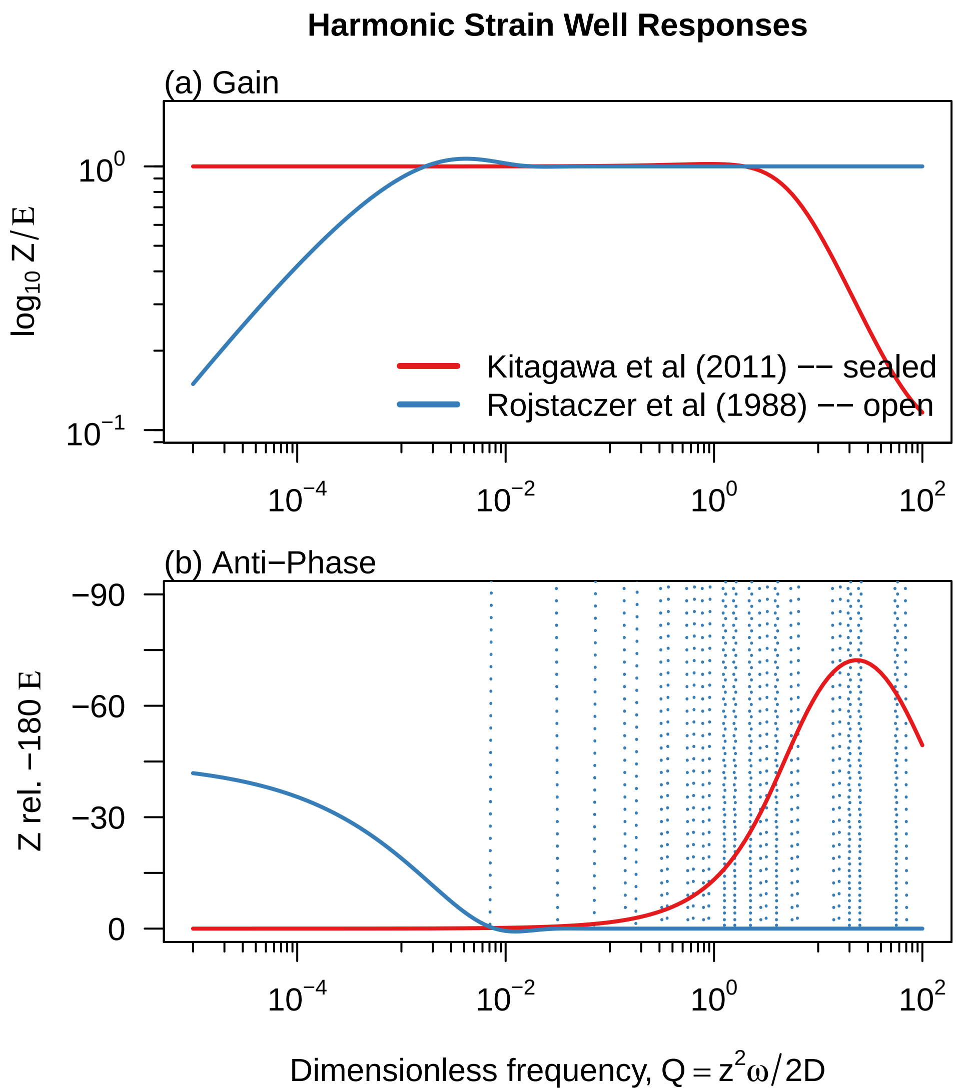


Figure 7: A comparison of well responses to harmonic strain. The phase of the water level is relative to $-180°$ the phase of strain.

5.2 Responses to pressure head (all open)

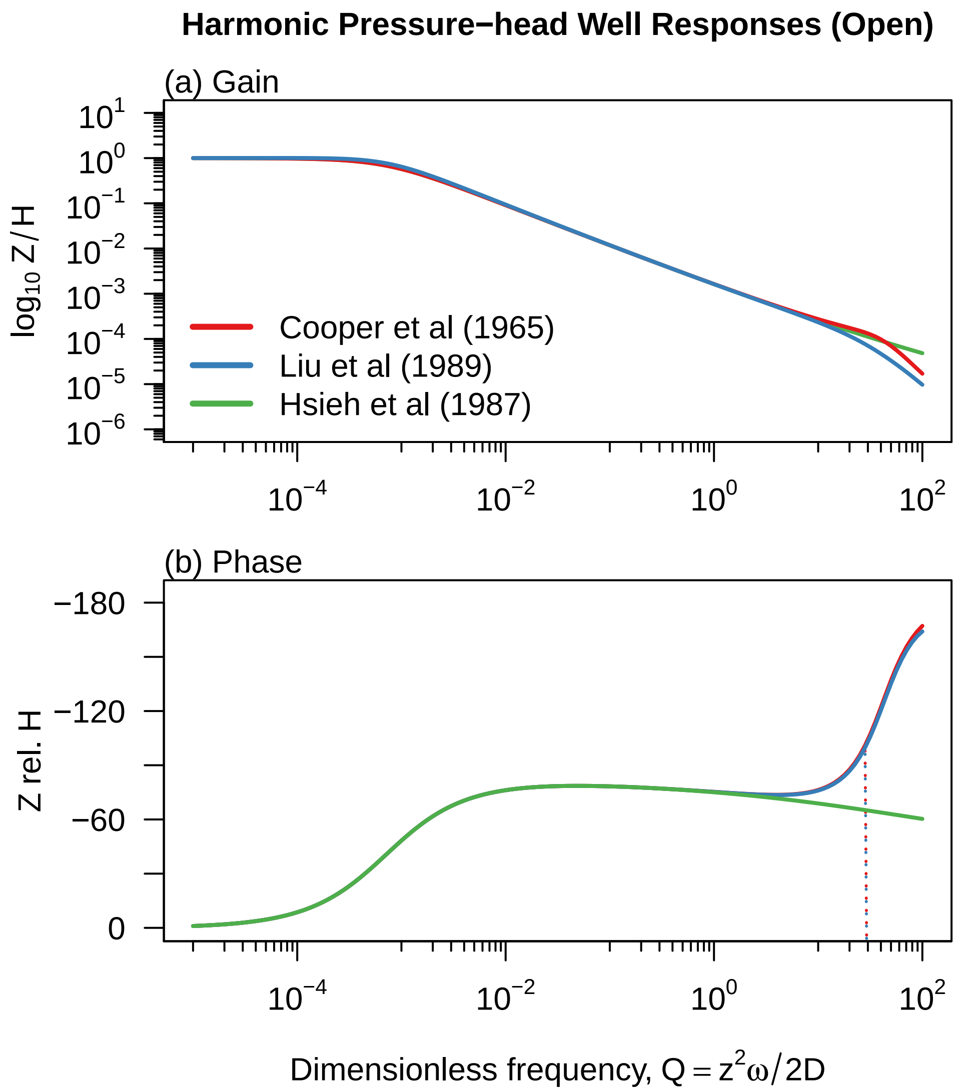


Figure 8: A comparison of well responses to harmonic pressure-head, from Cooper et al. (1965); Hsieh et al. (1987); Liu et al. (1989) (all for unsealed).

References

Cooper, H. H., Bredehoeft, J. D., Papadopulos, I. S., and Bennett, R. R. (1965). The response of well-aquifer systems to seismic waves. *Journal of Geophysical Research*, 70(16):3915–3926.

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